A Constraint to the Parity-Conserving Parameter of the Neutron-Antineutron Oscillations

M.I.Krivoruchenko
Institute for Theoretical and Experimental Physics,
B.Cheremushkinskaya, 25, 117259 Moscow, Russia
(e-mail: mikhail@vxitep.itep.ru)

Abstract

The phenomenology of neutrino oscillations is extended to the neutron-antineutron oscillations predicted by the grand unified theories. There are six parameters in general describing the one-flavor oscillations: two real Majorana masses (of opposite signs), three Euler's angles, and a phase multiplier χ. The sum of the Majorana masses, treated as a small parity-violating parameter ϵ, induces the nuclear decays $(A, Z) \rightarrow (A-2, Z)$ and determines the oscillation rate of neutrons in the vacuum. We derive a constraint to a small parity-conserving parameter ϵ' representing a combination of two Euler's angles. This parameter gives no significant contribution to the neutron oscillation rate in the vacuum, while contributes to the nuclear decay rate. The nuclear stability implies $|\epsilon'| < 1yr^{-1}$. Absence of the vacuum neutron oscillations together with existence of the nuclear decays $(A, Z) \rightarrow (A-2, Z)$ would become simplest manifestation of the $\epsilon' \neq 0$.

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Prediction[1] of the neutron-antineutron oscillations in the grand unified theories (GUT's) stimulated searches for the neutron-antineutron oscillations in the vacuum and in nuclei. The existing experiments constrain[1−10] the vacuum oscillation time to be $\tau = 1/ \mid \epsilon \mid > 1$ yr.

The phenomenology of the neutron oscillations is identical to the phenomenology of the one-flavor neutrino oscillations[11−14]. The neutron is described by a superposition of two real Majorana fields φ_k with different masses μ_k. The free neutron Lagrangian takes the form

$$L_f = \frac{1}{2} \sum_{k=1,2} \bar{\varphi}_k (i\hat{\nabla} - \mu_k)\varphi_k \tag{1}$$

where $\bar{\varphi}_k = \varphi_k \gamma_0$. The fields φ_1 and φ_2 are expressible in terms of the complex neutron field ψ as follows

$$\varphi_{kL} = a_k \psi_L + b_k \psi^*_L. \tag{2}$$

We remind that in the Majorana representation $\gamma^*_\mu = \tilde{\gamma}^\mu = -\gamma_\mu$, $\gamma^*_5 = \tilde{\gamma}_5 = -\gamma_5, \psi_c = \psi^*$. The left components are given by $\psi_L = \frac{1}{2}(1-\gamma_5)\psi$, etc. It follows then $\varphi_{kR} = (\varphi_{kL})^*$.

The kinetic term of the Lagrangian takes the standard form $L_{kin} = \bar{\psi} i \hat{\nabla} \psi$ for

$$\sum_{k=1,2} a_k a^*_k = \sum_{k=1,2} b_k b^*_k = 1, \sum_{k=1,2} a_k b^*_k = 0. \tag{3}$$

The mass term of the Lagrangian ($L_f = L_{kin} + L_m$) becomes

$$-L_m = m_D \bar{\psi}_R \psi_L + m_L \bar{\psi}_{cR} \psi_L + m_R \bar{\psi}_{cL} \psi_R + m^*_D \bar{\psi}_R \psi_L + m^*_L \bar{\psi}_L \psi_{cR} + m^*_R \bar{\psi}_R \psi_{cL} \tag{4}$$

where

$$m_D = \sum_{k=1,2} a_k b_k \mu_k, \; m_L = \sum_{k=1,2} a^2_k \mu_k, \; m^*_R = \sum_{k=1,2} b^2_k \mu_k. \tag{5}$$

The coefficients a_k and b_k can be interpreted as components of two orthogonal spinors. The general expressions for these coefficients are given by $a_k = D^{(1/2)}(\alpha, \beta, \gamma)_{k1} e^{i\chi}$ and $b_k = D^{(1/2)}(\alpha, \beta, \gamma)_{k2} e^{i\chi}$. The spin-1/2 rotation matrix has the form

$$D^{(1/2)}(\alpha, \beta, \gamma) = \begin{pmatrix} \cos(\frac{\beta}{2}) e^{i(\alpha+\gamma)/2} & \cos(\frac{\beta}{2}) e^{i(-\alpha+\gamma)/2} \\ -\sin(\frac{\beta}{2}) e^{i(\alpha-\gamma)/2} & \cos(\frac{\beta}{2}) e^{i(-\alpha-\gamma)/2} \end{pmatrix}, \tag{6}$$

the Euler's angles α, β, γ are defined as in Ref.15, Ch.58. The phase χ is introduced to generate a two-parameter family $\beta = \gamma = 0$ of the orthogonal spinors polarized in the z-direction.

The masses m_D, m_L, and m_R become

$$\begin{aligned} 2m_D e^{-i\chi} &= \sin\beta(\mu_1 e^{i\gamma} - \mu_2 e^{-i\gamma}), \\ 2m_L e^{-i(\alpha+\chi)} &= \cos^2(\tfrac{\beta}{2})\mu_1 e^{i\gamma} + \sin^2(\tfrac{\beta}{2})\mu_2 e^{-i\gamma}, \\ 2m_L^* e^{i(\alpha-\chi)} &= \sin^2(\tfrac{\beta}{2})\mu_1 e^{i\gamma} + \cos^2(\tfrac{\beta}{2})\mu_2 e^{-i\gamma}. \end{aligned} \tag{7}$$

Here $\bar{\psi}_{cR} = ((\psi_c)_R)^+\gamma_0$, etc. Given that complex masses m_D, m_L, m_R are known, one can find the diagonal masses μ_k, the Euler's angles α, β, γ, and the phase χ:

$$\begin{aligned} \tan\beta &= m_D/(m_L e^{-i\alpha} - m_R^* e^{i\alpha}), \\ \mu_k e^{i(\pm\gamma+\chi)} &= m_L e^{-i\alpha} + m_R^* e^{i\alpha} \pm \sqrt{(m_L e^{-i\alpha} - m_R^* e^{i\alpha})^2 + m_D^2}. \end{aligned} \tag{8}$$

The sign $(+)$ stands for $k = 1$. The values α, β, γ, and χ are fixed from a requirement that $\tan\beta$ and μ_k be real. The mass term of the Lagrangian contains tree complex masses m_D, m_L, and m_R and therefore six independent parameters. These parameters are described in terms of the two diagonal masses μ_k, three Euler's angles α, β, γ, and one phase χ. The phase transformation $\psi \to e^{-i\alpha/2}\psi$ can be made to set α equal to zero. The chiral transformation $\psi \to e^{i\gamma_5\chi/2}\psi$ removes the phase χ from Eqs.(7) and (8).

The Lagrangian (2) can finally be written in the form

$$L_f = \bar{\psi}(i\hat{\nabla} - m)\psi - \frac{1}{2}\Delta\bar{\psi}i\gamma_5\psi - \frac{1}{2}\epsilon\bar{\psi}_c\psi - \frac{1}{2}\epsilon^*\bar{\psi}\psi_c - \frac{1}{2}\epsilon'\bar{\psi}_c i\gamma_5\psi - \frac{1}{2}\epsilon'^*\bar{\psi}i\gamma_5\psi_c \tag{9}$$

where $m = (m_D + m_D^*)/2$, $\Delta = i(m_D - m_D^*)$, $\epsilon = m_L + m_R$, and $\epsilon' = i(m_L - m_R)$.

The choice of signs of the μ_k is a matter of convention. We assume $\mu_1 \approx -\mu_2 > 0$. It follows then that we are working in a region of small values $\mu_1 + \mu_2$, $\pi/2 - \beta$, γ and χ. To the lowest order in these parameters, we get

$$\begin{aligned} m &= \tfrac{1}{2}\sin\beta(\mu_1\cos(\gamma+\chi) - \mu_2\cos(\gamma-\chi)) \\ &\approx \tfrac{1}{2}(\mu_1 - \mu_2), \\ \Delta &= -\sin\beta(\mu_1\cos(\gamma+\chi) + \mu_2\cos(\gamma-\chi)) \\ &\approx -(\mu_1 - \mu_2)\chi, \\ \epsilon &= e^{i\alpha}\tfrac{1}{2}(e^{i\gamma}\cos^2(\tfrac{\beta}{2})(\mu_1 e^{i\chi} + \mu_2 e^{-i\chi}) + e^{-i\gamma}\sin^2(\tfrac{\beta}{2})(\mu_1 e^{-i\chi} + \mu_2 e^{i\chi})) \\ &\approx e^{i\alpha}\tfrac{1}{2}(\mu_1 + \mu_2), \\ \epsilon' &= ie^{i\alpha}\tfrac{1}{2}(e^{i\gamma}\cos^2(\tfrac{\beta}{2})(\mu_1 e^{i\chi} - \mu_2 e^{-i\chi}) - e^{-i\gamma}\sin^2(\tfrac{\beta}{2})(\mu_1 e^{-i\chi} - \mu_2 e^{i\chi})) \\ &\approx ie^{i\alpha}\tfrac{1}{2}(\mu_1 - \mu_2)(\pi/2 - \beta + i\gamma). \end{aligned} \tag{10}$$

In the external electromagnetic field, the neutron Lagrangian acquires a term $L_{int} = \bar{\psi}\frac{1}{2}\mu_n\sigma_{\mu\nu}F_{\mu\nu}\psi$, with μ_n being the neutron magnetic moment. The chiral transformation

$\psi \to e^{i\gamma_5\chi/2}\psi$ with $2m\chi = -\Delta$ removes the second term from Eq.(9), generating an interaction $L'_{int} = \bar{\psi}\frac{1}{2}d\sigma_{\mu\nu}\tilde{F}_{\mu\nu}\psi$ with $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\tau\upsilon}F_{\tau\upsilon}$. The parameter Δ is related to the neutron electric dipole moment $d = \mu_n\frac{\Delta}{2m}$. Limits to the dipole moment d are, however, many orders of magnitude lower then a value expected from the GUT's.

The vacuum oscillation rate to the lowest order is determined solely by parameter ϵ. The neutron oscillation time in the vacuum is $\tau = 1/|\epsilon|$.

The nuclear stability poses, however, comparable constraints to the ϵ and ϵ', since the nuclear decays are equally well induced by terms of the Lagrangian (9), proportional to the values ϵ and ϵ'. There is no interference between the corresponding terms, since they induce the opposite-parity transitions. The nuclear width can therefore be represented by a sum $\Gamma = \Gamma_- + \Gamma_+$. The value $\Gamma_- \propto |\epsilon|^2$ describes the parity-violating transitions, whereas the value $\Gamma_+ \propto |\epsilon'|^2$ describes the parity-conserving transitions. The first kind of the transitions is well known[1−10].

Shown in Fig.1 is a typical diagram contributing the nuclear decay $(A,Z) \to (A-2,Z)$ accompanied by emission of π-mesons. The parity-violating transitions differ from the parity-conserving transitions only by form of the vertex describing the neutron-antineutron annihilation. In the first case the vertex is proportional to the ϵ, whereas in the second case it is proportional to the $\epsilon' i\gamma_5$. The matrix $i\gamma_5$ mixes the upper and lower bispinor components. The energy release in the nuclear decay is high (about two nucleon masses), so such a mixing cannot produce a suppression $(v_F/c)^2$ with v_F being the Fermi velocity of nucleons, despite the decaying nucleus represents a nonrelativistic system. We get therefore

$$\Gamma_+/|\epsilon'|^2 \sim \Gamma_-/|\epsilon|^2. \tag{11}$$

Using estimates[1−8] for the parity-violating nuclear width Γ_- and results for testing the nuclear stability[10] we derive a constraint

$$|\epsilon'| < 1yr^{-1}. \tag{12}$$

An observation of the vacuum neutron oscillations would give an evidence for the $\epsilon \neq 0$. At the same time, an observation of the nuclear decays $(A,Z) \to (A-2,Z)$ together with zero or low vacuum oscillation rate of neutrons would give a direct experimental evidence for the $\epsilon' \neq 0$.

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